**SYLLABUS: HISTORY OF MATHEMATICS**
**CTY SUMMER PROGRAM**

Day 1: 1. Students make name signs, answer these questions: What do you prefer to be called? Have you done CTY before? If so, what course and topics? What is the last thing you remember studying in school (or in a distance learning program)? Why are you taking this class? What are you interested in learning? Do you have any concerns about the course?
2. Develop class rules together, talk about expectations, classroom procedures, and cooperative group work
3. Brainstorm: What is math?
4. Beginning of Mathematics Handout: Work with a partner to figure out how early people would have figured out how to tell time each day, the length of the year, etc. Call on pairs for their answers
5. Read about Ancient Calendars
6. Read about the Earliest Clocks
7. Pass out map of the ancient world, discuss
8. Who's Who on the Baseball Team: A logical thinking activity
9. Dominoes on a Checkerboard: A deductive reasoning question
10. Space for Ace: a puzzle.
11. Read "History of Counting"
12. Look at cartoon about "the preposterous googol"
13. Look at a handout on how to say the big numbers
14. Practice using various early systems of counting: Egyptian, Roman, Greek, Mayan
15. Do conversion problems with Egyptian unit fractions
16. Write fractions using Eye of Horus fractions, and answer the question: Can any fraction be written using Eye of Horus fractions?

Day 2: 1. How large is your yard? Measure body parts with a partner, to see what size your Egyptian units are.
2. Write a story about something going wrong because of discrepancies in measurement
3. Read "Taxes and Triangles."
4. Construct a perpendicular bisector.
5. Practice using Mayan calendar numbers
6. Practice using Mayan numerals
7. Practice using Chinese Shang Numerals
8. Stars Around the Moon: finding the pattern
9. Read about Thales in "Mathematicians are People, Too"
10. Find the heights of objects around campus using Thales' method

Day 3: 1. Prove Pythagorean theorem using the Chinese method, discuss math developed in different areas
2. Read about Pythagoras in "Mathematicians are People, Too"
3. Read poem for two voices about Radicals

4. Make Platonic solids using toothpicks and gum drops
5. Make paper puzzles, figure out how to put them together to form a tetrahedron and a cube.
6. Draw pictures of collapsed solids
7. Pass out map of Greece
8. Write several terms, including the nth term for triangular, square, and oblong numbers
9. Find patterns in figurate families: Triangular through Octagonal
10. Find Pythagorean discoveries (relationships between various figurate numbers)
11. Cut up a figure to prove the Pythagorean theorem in a different way
12. Experiment with Pythagorean triplets
13. Find ratios of sequential Fibonacci numbers, discuss convergence
14. Discuss the golden mean, look at examples in art
15. Construct a golden spiral inside a golden rectangle
16. Divide a circle into 3 parts and then 6 parts, using a compass
17. Read "The Stargazers" from "String, Straightedge, and Shadow"

Day 4: 1. Work with Babylonian numbers
2. Read a poem for two voices on Zero
3. Construct golden triangles, pentagram
4. Show how to find phi using the quadratic formula
6. Find approximations for phi using square roots, and using infinite continued fractions
4. Read about Euclid in "Mathematicians are People, Too"
5. Read about Euclid's Axioms
6. Use Euclid's Algorithm
7. Find the relationship between numbers, their Greatest Common Divisor, and Least Common Multiple
8. Find patterns in various sizes of painted cubes (how many have 3 white faces, how many have 2, 1, or no white faces)
9. Find patterns in Perfect Numbers
10. Read "Square and Circle"
11. Prove $ab = ((a + b)/2)^2 – ((a – b)/2)^2$
12. Read Pi poem for two voices
13. Read "And the Pi goes on," from "More Joy of Mathematics."
14. Read "Fun and Interesting Facts about Pi"
15. Read about Archimedes in "Mathematicians are People, Too"
16. Go outside, and estimate Pi by finding the circumference and diameter of various circular objects on campus
17. Archimedes' Mobiles: Find the weights of different parts of a mobile
18. A Balancing Act: Find the center of gravity on triangles
19. Find Archimedes' Law of the Lever

Day 5: 1. Answer question: Is math a science, a language, or a way of thinking?
2. Answer questions: What are your most favorite parts of class, least favorite parts of class, and what are you still curious about?
3. Do a skit based on Archimedes, "The King's Crown"
4. Use the Sieve of Eratosthenes, and answer questions about prime numbers
5. Read about the largest known prime number
6. Complete Magic Squares made up of prime numbers
7. Odds in the Making: finding primes and powers of two that sum to form an odd number
8. Is It Always True? Verifying Goldbach's conjecture for some even numbers
9. Use Heron's Square Root Method
10. Use Heron's Formula
11. Read about Hypatia in "Mathematicians are People, Too"
12. Read handout on conic sections, identify conic sections, and make conic sections by cutting playdough with a string
13. Form a parabola, an ellipse, and a hyperbola using paper folding
14. A Stamp Stumper: a modern day version of Diophantine problems
15. Read about the abacus
16. Build an abacus with smarties on a frame drawn on paper
17. Represent various numbers using the abacus
18. Practice addition and subtraction using the abacus
19. Read about Napier in "Mathematicians are People, Too"
20. Make Napier's bones, and practice calculations using Napier's bones

Day 6: 1. Answer question: What qualities make a good mathematician?
2. Do a skit based on Napier, "The Magic Rooster"
3. Read about Omar Khyam in "Mathematicians are People, Too"
4. Read about Fibonacci in "Mathematicians are People, Too"
5. Read a handout on the evolution of Hindu-Arabic numerals
6. Read about Fibonacci in nature, look at a pineapple
7. Fibonacci Discoveries: finding patterns when the squares of Fibonacci numbers are added
8. Cut out 13 by 13 square out of graph paper, cut into pieces to form a 8 by 21 square, discuss why the areas appear to be different
9. Do a magic trick with Fibonacci-like numbers: the $7^{th}$ number times 11 is equal to the sum of the first 10. Prove that this is true.
10. Read about Cardano in "Mathematicians are People, Too"
11.. Read about Galileo in "Mathematicians are People, Too"
12. Galileo Drops the Ball: finding the relationship between time and velocity
13. Have Gravity: Must Travel: finding the relationship between time and distance
14. Heartbeats and Pendulums: finding the relationship between pendulum length and time of swing
15. Read about Descartes in "Mathematicians are People, Too"
16. Plot and Swat: Connect the coordinates to form a fly
17. Area the Easy Way: finding area of polygons using their vertices

18. Read about "The Cycloid: The Helen of Geometry" from "The Joy of Mathematics"
19. Read about Fermat in "Mathematicians are People, Too"
20. Primes and Squares: finding the pattern in which prime numbers can be expressed as the sum of exactly two perfect squares
21. The Proof is in the Pudding: a logic puzzle
22. Suspicious Sailors: a logic puzzle
23. Work on posters, constructing your own number system. Posters will include a description of the imaginary civilization that developed this system why it developed as it did; a comparison of the system to other systems we have studied; an analysis of its characteristics including whether it includes a zero, what base it is, whether it is grouping, repetitive, uses place value, is additive, or is subtractive; the symbols used to represent numbers; and sample calculations of addition and subtraction.

Day 7: 1. Do a skit based on Descartes, "The Best Put to the Test"
2. Play a game: Find the Hurkle
3. Play a game: Where's the Rectangle?
4. Read handout on the arithmetic triangle, describing its discovery in China. Translate Rod numerals into Hindu-Arabic numbers, add two more rows to the triangle, find three patterns in the triangle
5. Discuss patterns students found in the triangle
6. Lead students through any of these patterns not mentioned: 1s, consecutive integers, triangular numbers, Fibbonacci numbers, powers of two, evens and odds
7. Show students about how powers of (a + b)^n show up in the triangle
8. Students find all the combinations of flipping four coins, in a systematic manner. Discover this represented in the triangle.
9. Read about Pascal in "Mathematicians are People, Too"
10. Read about Newton in "Mathematicians are People, Too"
11. Explore how to find the area under a curve using rectangles
12. Look at handout depicting Seki Kowa's work on Calculus in Japan
13. Read about Euler in "Mathematicians are People, Too"
14. Solve the Bridges of Konigsberg problem
15. Work problems to practice finding whether networks are traversable. Discover pattern in which networks are traversable
16. Vertices, Regions, and Arcs: finding a formula that relates these
15. Discover the relationship between Faces, Vertices, and Edges in regular polyhedra
16. Knight's Move on the Chessboard: visit every square on the chessboard exactly once using the knight's move
17. Continue work on Number System posters

Day 8: 1. Write one page on: Who is your favorite mathematician we have studied so far, and why?
2. Read about Agnesi in "Mathematicians are people, Too"

3. Graph “The Witch of Agnesi”
4. Exploring Isoperimetric Figures: figure out which figure has the largest area
5. “X’s, Y,s and Zzzzzzz’s” a skit based on Maria Agnesi
6. Read about Banneker in “Mathematicians are People, Too”
7. A Clockmaker’s Challenge: figuring out the number of revolutions gears move, and the directions they turn
8. Puzzles from Benjamin Banneker’s Collection
9. A collection of Puzzlers
10. Putting on your logic hat
11. Read about Lagrange in “Mathematicians are People, Too”
12. Lagrange’s Four Square Theorem
13. The Metric Highway, discovering the relationship between miles and kilometers
14. Metric Crossword
15. Logic problem: Discover the next part of the pattern, for 9 different patterns
16. Continue work on number system

Day 9: 1. Read about Sophie Germain in “Mathematicians are People, Too”
2. Counting Divisors: discover how exponents can be used to determine the number of divisors for any number
3. Work with Happy Numbers
4. Pose the problem of adding together the numbers from 1 to 100, ask for clever ways of doing this.
5. Read about Gauss in Mathematicians are People, Too”
6. What’s for Lunch? using modular arithmetic
7. Make Tangrams and figure out how to make various shapes using the Tangrams
8. Make an egg shape and cut into pieces, create birds
10. Experimenting with Mobius strips: Students make hypotheses and then find out how many sides each of the following have, and what happens when you cut them:
    A) Basic Mobius strip
    B) Mobius strip cut 1/3 of the way across
    C) 2 half twists
    D) 3 half twists
    E) Double Mobius strip
    F) Triple Mobius strip
    G) Plus shape
11. Read a poem about the Mobius strip
12. Read about applications of the Mobius strip
13. Read about the Mobius strip in Star Trek
14. Discuss Klein bottle
15. Look at three rings and determine whether they are linked or not
16. Read about knots
17. Give an introduction to topology, using pictures and playdough
18. Students determine whether various objects are topologically equivalent

19. Inside or Outside?: Students determine the method for determining whether points are inside or outside a simple closed curve.
20. Continue work on number system

Day 10: 1. Read about Mary Somerville in "Mathematicians are People, Too"
2. Star Shapes: Use Pick's Theorem to determine the areas of unusual figures
3. Read about Babbage in "Mathematicians are People, Too"
4. Checkerboard Challenge: find the number of squares of all sizes on an 8 by 8 checkerboard, by figuring out the pattern for smaller size checkerboards
5. Read about Lovelace in "Mathematicians are People, Too"
6. Calculator Fun: finding word answers to clues written upside-down on the calculator
7. Translate numbers to binary, make binary pictures, and determine whether various binary numbers are symmetric or non-symmetric
8. Write programs to get out of mazes
9. Read about mazes
10. Read about Abel in "Mathematicians are People, Too"
11. Read about Kovalevsky in "Mathematicians are People, Too"
12. Predictable Patterns: continue sequences
13. The Chessboard covered with Wheat: Complete a table to find the total number of grains on all 64 squares
14. Four fours: Use exactly 4 4s and the operations +, -, *, and /, to make equations that equal 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
15. Read about Charles Dodgson
16. Lighthearted Logic, and More Logic Puzzles, by Charles Dodgson
17. Dodgson's Doublet Puzzles
18. Make a triflexagon, a tri-tetra flexagon, and a hexa-hexa flexagon
19. Present number system

Day 11: 1. Read about Galois in "Mathematicians are People, Too"
2. Read about and examine a variety of impossible figures and optical illusions
3. Make a moving illusion
4. Discuss illusions at Disneyland, in movies, etc.
5. Read about Kovalevsky
6. Read about set theory
7. Read about the Hotel Infinity
8. Begin to work on a skit on a famous mathematician: skit must cover the front and back of at least one page, be on a mathematician we have studied, and involve all members of the group.

Day 12: 1. Read about Emmy Noether in "Mathematicians are People, Too"
2. Work with groups using handout from "Women & Numbers"
3. A Puzzling Mystery: A word jumble based on Emmy Noether
4. Make a square and form the group chart for rotation and reflection of a square
5. Do dissection problems: Cutting up mathematics and Squaring up
6. Read about Ramanujan in "Mathematicians are People, Too"

7. Counting partitions
8. The curious cab: finding the two sets of cubes that sum to 1729.
9. The Snowflake Curve: finding a formular for the number of sides, length of each side, and perimeter of the snowflake, at stage n.
10. Follow a rule to make a fractal
11. Read about fractals: dragon, snowflake, anti-snowflake, Sierpinsky triangle
12. Create your own fractal
13. Read about fractals and their dimension, and their applications: the Peano curve and population, clouds, ferns, cinematography, astronomy, economics, meteorology, ecology.
14. Read about Venn
15. Problem Solving using Venn diagrams, Venn Diagrams Solve the Problem
16. Who's In Charge: a logic puzzle
17. Crossing the River: the Wolf, Goat, Cabbage problem
18. Rowing Relay: a logic puzzle
19. Continue working on a skit on a famous mathematician

Day 13: 1. Read about Einstein in "Mathematicians are People, Too"
2. Read about Polya in "Mathematicians are People, Too"
3. Brainstorm problem-solving techniques
4. The Locker Problem
5. Pouring with Pails: How can you bring up from the river exactly six quarts of water when you have only two containers, a nine quart pail and a four quart pail, to measure with?
6. Building Blocks: Use any three of the four numbers 8, 5, 40, 10, to create true mathematical sentences, (solutions of 9, 13, 16, 3, 7, 0, 10, 1, 25 are given), inserting +, -, *, or / where needed.
7. Knight Pairs on the Chessboard: look at small cases to find the pattern, then determine the number of knight pairs on an 8 by 8 board, and an n by n board
8. Read about the perfect palindrome
9. Work with palindromes
10. Practice skits

Day 14: 1. Do a crossword puzzle reviewing the mathematicians we have studied
2. Read about Escher and the art of the Alhambra
3. Learn about friezes and their symmetries
4. Classify friezes as one of the 7 types
5. Figure out which regular polygons can tessellate the plane, and why
6. Practice using reflection and rotation on some sample problems to make Escher-like patterns
7. Draw figures that are symmetric about dashed lines, determine the number of rotation symmetries on figures, and the number of lines of symmetry
8. Create famous mathematician cards, and play "Go Mathematician." Students must request the card using a characteristic of the mathematician: for example, do you have "measured pyramids using shadows?"

9. Discover how many colors are necessary to color any map
10. Color maps
11. Read about codes and ciphers
12. Read about breaking “unbreakable” ciphers
13. Decode some sample codes
14. Practice skits

Day 15: 1. Answer questions students have asked on the “curious board”
2. Look at Imagine magazine on mathematics, including articles on fractals and codes
3. Look at recreational mathematics books
4. Practice skits one last time
5. Perform skits
6. Hand out list of books to read for further information on the History of Math
7. Hand out list of further summer programs in mathematics, math websites, and math organizations