

Detailed Course Information:

There are eleven weeks of topics and one week for the final exam. The following lists all 11 topics together with sample problems:

Chapter 1: The Pigeon Hole Principle

Among six persons there are always three who all know each other or three none of which knows each other.

Chapter 2: The Invariance Principle

It is known that an ordinary 8×8 chessboard can be covered by 1×2 tiles in $2^4 \cdot 901^2 = 12,988,816$ ways. Now let us cut out two diagonally opposite corners of an ordinary chessboard. In how many ways can you tile the remaining 62 squares with 31 1×2 tiles?

Chapter 3: The Extremal Principle

Find all integral solutions to $a^2 + b^2 + c^2 = a^2b^2$. (USAMO 1976.3)

Chapter 4: The Induction Principle

Given that z is a complex number such that $z + 1/z = 2 \cos(3^\circ)$, find the least integer that is greater than $z^{2000} + 1/z^{2000}$. (AIME 2000II.9)

Chapter 5: Systematic Checking

You have only 5 cent and 7 cent stamps. What postage amounts are you able to make? Can you generalize? (ARML 1995)

Chapter 6: Number Theory I (Basics)

In the expansion of $(ax + b)^{2000}$ where a and b are relatively prime positive integers the coefficients of x^2 and x^3 are equal. Find $a+b$. (AIME 2000I.3)

Chapter 7: Number Theory II (Vieta and Vieta Jumping)

If a, b are positive integers and $(ab + 1)$ divides $(a^2 + b^2)$ then $(a^2 + b^2) / (ab + 1)$ is a square. (IMO 1988.6)

Chapter 8: Combinatorics I (Basics)

A game uses a deck of n different cards where n is an integer with $n \geq 6$. The number of possible sets of 6 cards that can be drawn from the deck is 6 times the number of possible sets of 3 cards that can be drawn. Find n . (AIME 2005II.1)

Chapter 9: Combinatorics II (Combinatorial Geometry)

The surface of a soccer ball consists of only black pentagons and white hexagons. The edges of all pentagons border only on hexagons while the edges of the hexagons border alternately on pentagons and hexagons. Determine the number of pentagons and hexagons of the soccer ball. (Germany 1983)

Chapter 10: Sequences

The sequence a_n is defined by $a_0 = 1$, $a_1 = 1$ and $a_n = a_{n-1} + a_{n-1}^2 / a_{n-2}$.
The sequence b_n is defined by $b_0 = 1$, $b_1 = 3$ and $b_n = b_{n-1} + b_{n-1}^2 / b_{n-2}$.
Find b_{32}/a_{32} . (AIME 2008II.6)

Chapter 11: Polynomials

Two roots of the real quartic $x^4 - 18x^3 + ax^2 + 200x - 1984 = 0$ have product -32 . Find a .
(USAMO 1984.1)